# Chapter 37 – Public Goods

## What is a "public good"?

→ Public goods have two attributes that make them different from private goods:

① They are non-excludable

→ non-excludable means you can't exclude people who don't pay

e.g. even if you don't pay to stop global climate change, we can't exclude you from the benefits

② They are non-rival

→ non-rival means that one person's use of the good doesn't affect another person's use of the good.

e.g your enjoyment of clean air doesn't impact other people enjoying the clean air

## When to provide a public good?

→ start w/ discrete good case since it's simplist

→ Example: 2 roommates deciding to buy a TV.

The TV is a public good b/c it is non-rival (both can watch and enjoyment not less) and non-excludable (it'll be put in the common living room they share.

→ let $w_1$ and $w_2$ be each roommates initial wealth

→ $g_1$ and $g_2$ are their contributions to the TV purchase

→ $x_1$ and $x_2$ give money leftover for private consumption.

⇒ budget constraints are:

$$x_1 + g_1 = w_1$$

$$x_2 + g_2 = w_2$$

→ to buy TV of cost $c$, it must be that $g_1 + g_2 \geq c$

→ Utility:

$u_1(x_1, G)$, $u_2(x_2, G)$

$G$ = value of public good, the TV

→ consumed in same amount by both

→ $G = 1$ or $G = 0$ if TV present or not

→ can find reservation prices:

$$u_1(\underbrace{w_1 - r_1}, 1) = u(w_1, 0)$$

$w_1 - r_1 = x_1$

this equation defines $r_1$, reservation price for person 1

$\rightarrow \underbrace{u_2(w_2 - r_2, 1) = u_2(w_2, 0)}_{\text{defines } r_2}$

→ note how reservation price depends on wealth - willingness to pay depends on ability to pay

→ want to compare utility of roommates under 2 situations:

1. No TV : $(\underbrace{w_1, w_2}_{\text{private cons.}}, \overbrace{0}^{\text{public cons}})$

2. TV : $(x_1, x_2, 1)$

$$x_1 = w_1 - g_1$$
$$x_2 = w_2 - g_2$$

Buying TV is a Pareto improvement over not buying when both better off w/ buying than w/o:

$$u_1(w_1, 0) < u_1(x_1, 1)$$
$$u_2(w_2, 0) < u_2(x_2, 1)$$

using reservation prices:

$$u_1(w_1 - r_1, 1) = u_1(w_1, 0) < u_1(x_1, 1) = u(w_1 - g_1, 1)$$
$$u_2(w_2 - r_2, 1) = u_2(w_2, 0) < u_2(x_2, 1) = u_2(w_2 - g_2, 1)$$

contrib to pub good

$\Rightarrow$ Pareto improvement if:

$$u_1(w_1 - r_1, 1) < u_1(w_1 - g_1, 1)$$

$$u_2(w_2 - r_2, 1) < u_2(w_2 - g_2, 1)$$

~~$\Rightarrow$ if private cons~~

$\Rightarrow$ (b/c private cons. ↑ utility):

$$\cancel{w_1} - r_1 < \cancel{w_1} - g_1$$

$$\cancel{w_2} - r_2 < \cancel{w_2} - g_2$$

$$\Rightarrow \quad r_1 > g_1$$

$$r_2 > g_2$$

→ makes sense ⇒ buy good if willingness to pay ~~for both each~~ is less than contribution for each of the roommates

⇒ note this condition also means that the sum of each roommates willingness to pay will exceed the total cost:

$$r_1 + r_2 > g_1 + g_2 = c$$

Lessons from this:

1. If sum of WTP > total cost, can always find a payment scheme where both better off w/ public good than w/o

2. Whether Pareto efficient to provide good or not depends on initial distribution of wealth – since reservation prices / WTP depend on wealth
   → exception are quasilinear preferences

Private provision of a public good

→ Think about the problem btwn the 2 roommates – do I contribute to the purchase of the TV or not?

→ Let's suppose that $w_1 = w_2 = \$500$ and that $r_1 = r_2 = \$100$ the cost of the TV is \$150, so

$r_1 + r_2 = \cancel{2055}\ \$200 > \$150 = \text{cost of TV}$

⇒ Pareto efficient to provide the TV.

→ But their decisions independently can be represented by the following normal form game:

Roommate 2

| | | Buy | Don't Buy |
|---|---|---|---|
| Roommate 1 | Buy | -50, -50 | -50, 100 |
| | Don't Buy | 100, -50 | 0, 0 |

PSNE → no one buys the TV

Milton Friedman: "The problem w/ public goods is not that one person pays for what someone else gets, but that nobody pays and nobody gets, even though the good is worth more than it costs to produce."

→ Note that roommates might be able to reach agreement where ~~both~~ one buys and other pays him some amount.

→ ~~There are gains to~~ be

→ But as more roommates/people benefitting from the public good, harder to reach these agreements.

→ This result is called the "free-rider problem"

→ people value the public good at more than its cost, but everyone hopes someone else will provide the good

→ This results in a market failure - the free market underprovides public goods

## Providing different levels of a public good

→ now consider the question of how much of a public good to provide

→ consider example of before w/ same notation except now we let $G$ = the quality of the TV, a continuous value

→ the roommates face the constraint:

$$\underbrace{x_1 + x_2 + c(G)}_{\text{total resources expended on private and public goods}} = \underbrace{w_1 + w_2}_{\text{total resources they have}}$$

→ The problem of finding the Pareto efficient allocation can be written as maximizing the utility of one of the roommates while holding the other's utility constant:

$$\max_{x_1, x_2, G} \quad u_1(x_1, G)$$

$$\text{s.t.} \quad u_2(x_2, G) = \bar{u}_2$$

$$\text{and} \quad x_1 + x_2 + c(G) = w_1 + w_2$$

Set up Lagrangian:

$$\mathcal{L} = u_1(x_1, G) - \lambda\left[u_2(x_2, G) - \bar{u}_2\right] - \mu\left[\underbrace{x_1 + x_2}_{\text{prices of private goods} = 1} + c(G) - w_1 - w_2\right]$$

FOCs:

① $\dfrac{\partial \mathcal{L}}{\partial x_1} = \dfrac{\partial u_1(x_1, G)}{\partial x_1} - \mu = 0$

② $\dfrac{\partial \mathcal{L}}{\partial x_2} = -\lambda \dfrac{\partial u(x_2, G)}{\partial x_2} - \mu = 0$

③ $\dfrac{\partial \mathcal{L}}{\partial G} = \dfrac{\partial u_1(x_1, G)}{\partial G} - \lambda \dfrac{\partial u_2(x_2, G)}{\partial G} - \mu \dfrac{\partial c(G)}{\partial G} = 0$

① $\Rightarrow \dfrac{\partial u_1(x_1, G)}{\partial x_1} = \mu$

② $\Rightarrow -\dfrac{\partial u_2(x_2, G)}{\partial x_2} = \dfrac{\mu}{\lambda}$

③ $\Rightarrow \dfrac{\partial u_1(x_1, G)}{\partial G} - \dfrac{\lambda \partial u_2(x_2, G)}{\partial G} = \mu \dfrac{\partial c(G)}{\partial G}$

$\Rightarrow \dfrac{1}{\mu} \dfrac{\partial u_1(x, G)}{\partial G} - \dfrac{\lambda}{\mu} \dfrac{\partial u_2(x_2, G)}{\partial G} = \dfrac{\partial c(G)}{\partial G}$

$\Rightarrow \dfrac{\partial u_1(x, G)}{\partial G} \Big/ \dfrac{\partial u_1(x_1, G)}{\partial x_1} + \dfrac{\partial u_2(x_2, G)}{\partial G} \Big/ \dfrac{\partial u_2(x_2, G)}{\partial x_2} = \dfrac{\partial c(G)}{\partial G}$

$\rightarrow |MRS_1| + |MRS_2| = MC(G)$

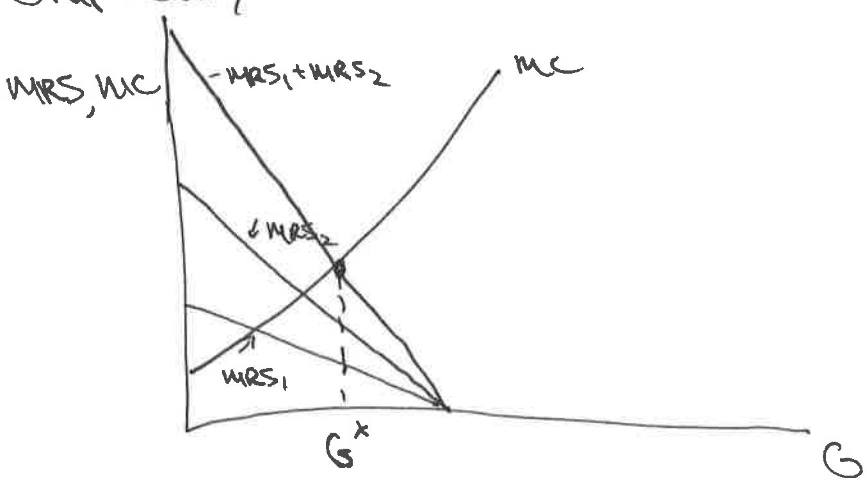
What does this mean?

→ that rate at which agents give up private consumption for public good consumption, summed over all agents, must equal the marginal cost of the public good at an optimum

→ think @ if this didn't hold → then, if, $|MRS_1 + MRS_2| < MC(G)$ society would be better off if spent less on public good and gave money to one or both agents to consume.

→ same reasoning if $|MRS_1| + |MRS_2| > MC(G)$

Graphically:



e.g. $u_1(x_1, G) = \ln(x_1) + 2\ln(G)$

$u_2(x_2, G) = \ln(x_2) + 2\ln(G)$

$c(G) = 4G$

$$\mathcal{L} = \ln(x_1) + 2\ln(G) + \lambda[\ln(x_2) + 2\ln(G) - \bar{u}_2] - \mu[x_1 + x_2 + c(G) - w_1 - w_2]$$

FOCs:

① $\frac{1}{x_1} - \mu = 0$

② $-\frac{\lambda}{x_2} - \mu = 0$

③ $\frac{2}{G} - \frac{\lambda 2}{G} - \mu \underbrace{\frac{\partial c(G)}{\partial G}}_{=4} = 0$

① $\Rightarrow$ $\frac{1}{x_1} = \mu$

② $\Rightarrow$ $\frac{1}{x_2} = \frac{-\mu}{\lambda}$

③ $\Rightarrow$ $\frac{2}{G} - \frac{\lambda 2}{G} = \mu 4$

$3G = \frac{1}{2}(w_1 + w_2)$

$\Rightarrow G = \frac{1}{6}(w_1 + w_2)$

$\Rightarrow$ $\frac{2x_1}{G} + \frac{2x_2}{G} = 4$

$\frac{2}{G}(x_1 + x_2) = 4$

$\Rightarrow$ $G = \frac{1}{2}(x_1 + x_2)$

$\rightarrow$ note, $x_1 + x_2 = w_1 + w_2 - c(G)$

$\Rightarrow G = \frac{1}{2}(w_1 + w_2 - 4G)$

$G = \frac{1}{2}(w_1 + w_2) - 2G$

Private Provision of the ~~Optimal~~ Optimal Level of the Public Good?

→ The free rider problem still exists
⇒ private provision will underprovide public goods

→ Each person will consider supply some of the public good herself, while forecasting how much of the public good others will provide.

→ consider simple example w/ 2 people, $c(G) = 4G$
and price of $x_1$ = price of $x_2 = 1$
→ let $g_1, g_2$ be contributions to public good of persons 1 and 2, respectively.
→ $G = \frac{g_1 + g_2}{4}$
⇒ person 1's problem:

$$\max_{x_1, g_1} u_1\left(x_1, \frac{g_1 + \bar{g}_2}{4}\right)$$

↑ amt person 1 thinks person 2 will contribute

$$\text{s.t. } x_1 + g_1 = w_1$$
$$\text{and } g_1 \geq 0$$

$$\mathcal{L} = u_1\left(x_1, \frac{g_1 + \bar{g}_2}{4}\right) - \lambda[x_1 + g_1 - w_1] + \mu[g_1]$$

① $$\frac{\partial \mathcal{L}}{\partial x_1} = \frac{\partial u_1(x_1, \frac{g_1 + \bar{g}_2}{4})}{\partial x_1} - \lambda = 0$$

② $$\frac{\partial \mathcal{L}}{\partial g_1} \quad \frac{\partial u_1(x_1, \frac{g_1 + \bar{g}_2}{4})}{\partial g_1} - \lambda + \mu = 0$$

③ $\frac{\partial \mathcal{L}}{\partial \lambda}$ : $x_1 + g_1 = w_1$ or $\lambda = 0$

④ $\frac{\partial \mathcal{L}}{\partial \mu}$ = $\mu > 0$ or $g_1 > 0$

$$\frac{(a)}{(b)} \Rightarrow \underbrace{\frac{\dfrac{\partial u_1(x_1, g_1+\bar{g}_2)}{\partial g_1}}{\dfrac{\partial u_1(x_1, g_1+\bar{g}_2)}{\partial x_1}}}_{MRS_1} = \frac{\lambda+\mu}{\lambda}$$

$\Rightarrow$ if $g_1 > 0$, then

$|MRS_1| = 1$

if $g = 0$, $|MRS_1| > 1$

→ can't take contributions away from public good

→ at corner sol'n where give zero

→ problem depends on how other responds

(e.g. $\bar{g}_2$)

consider specific example:

$$u_1 = \ln(x_1) + 2\ln(G)$$

$$u_2 = \ln(x_1) + 2\ln(G)$$

agent 1's maximization problem yields:

$$\frac{2}{g_1+\bar{g}_2} \cdot \frac{x_1}{1} = 1$$

$$\Rightarrow \frac{2x_1}{g_1+\bar{g}_2} = 1 \Rightarrow 2x_1 = (g_1+\bar{g}_2)$$

The agent's budget constraint says:

$$x_1 + g_1 = w_1$$

$$\Rightarrow x_1 = w_1 - g_1$$

$$\Rightarrow 2x_1 = \left(\frac{g_1 + \bar{g}_2}{4}\right)$$

$$\Rightarrow 2(w_1 - g_1) = \frac{g_1 + \bar{g}_2}{4}$$

$$8w_1 - 8g_1 = g_1 + \bar{g}_2$$

$$\Rightarrow 9g_1 = 2w_1 - \bar{g}_2$$

$$g_1 = \frac{2}{9}w_1 - \frac{\bar{g}_2}{9}$$

→ agent 2 solves same problem w/ same utility function ⇒ his sol'n the same form!

→ this defines how #2 responds

$$g_2 = \frac{2}{9}w_2 - \frac{\bar{g}_1}{9}$$

$$\Rightarrow g_2 = \frac{2}{9}w_2 - \left(\frac{2}{9}w_1 - \frac{g_2}{9}\right)\frac{1}{9}$$

$$g_2 = \frac{2}{9}w_2 - \frac{2}{81}w_1 + \frac{g_2}{81}$$

$$\Rightarrow g_2 - \frac{1}{81}g_2 = \frac{2}{9}w_2 - \frac{2}{81}w_1$$

$$\frac{81}{80}\cdot\frac{80}{81}g_2 = \left(\frac{2}{9}w_2 - \frac{2}{81}w_1\right)\cdot\frac{81}{80}$$

$$g_2 = \frac{1}{3}\cdot\frac{9}{8}w_2 - \frac{9}{8}\cdot\frac{2}{9}w_1$$

$$g_2 = \frac{3}{4}w_2 - \frac{1}{4}w_1$$

$$\Rightarrow g_1 = \frac{2}{3}w_1 - \frac{3}{4}\cdot\frac{1}{3}w_2 + \frac{1}{4}\cdot\frac{1}{3}w_1 = \frac{2}{3}w_1 - \frac{1}{4}w_2 + \frac{1}{12}w_1$$

$$\Rightarrow g_2 = \frac{\cancel{2}^{1}}{\cancel{9}}\cdot\frac{\cancel{81}^{9}}{\cancel{80}_{40}} w_2 - \frac{\cancel{2}^{1}}{\cancel{81}}\cdot\frac{\cancel{81}}{\cancel{80}_{40}} w_1$$

$$g_2 = \frac{9}{40} w_2 - \frac{1}{40} w_1$$

$$\Rightarrow g_1 = \frac{2}{9} w_1 - \frac{g_2}{9} = \frac{2}{9} w_1 - \frac{1}{9}\left(\frac{9}{40} w_2 - \frac{1}{40} w_1\right)$$

$$= \frac{2}{9} w_1 - \frac{1}{40} w_2 + \frac{1}{9 \cdot 40} w_1$$

$$= \left(\frac{2}{9} + \frac{1}{360}\right) w_1 - \frac{1}{40} w_2$$

$$= \frac{81}{360} w_1 - \frac{1}{40} w_2$$

$$g_1 = \frac{9}{40} w_1 - \frac{1}{40} w_2$$

$$\Rightarrow \frac{g_1 + g_2}{4} = \frac{G}{\cancel{4}} = \frac{1}{4}\left[\underbrace{\left[\frac{9}{40} w_1 - \frac{1}{40} w_2\right]}_{g_1} + \underbrace{\left[\frac{9}{40} w_2 - \frac{1}{40} w_1\right]}_{g_2}\right]$$

$$= \frac{1}{4}\left[\left[\frac{9}{40} w_1 - \frac{1}{40} w_1\right] + \left[\frac{9}{40} w_2 - \frac{1}{40} w_2\right]\right]$$

$$= \frac{1}{4}\left[\frac{8}{40} w_1 + \frac{8}{40} w_2\right]$$

$$\frac{G}{\cancel{4}} = \frac{1}{4}\frac{1}{5}\left[w_1 + w_2\right]$$

$$\underline{G = \frac{1}{20}\left[w_1 + w_2\right]}$$

under provided - optimal and was $\frac{1}{2}[w_1 + w_2]$

→ private provision of public goods doesn't work well

→ so how provide?

3 ways we'll mention:

1. Command - a central planner taxes and uses revenue to provide
   → how know optimal/desired amt?

2. Voting
   → people get to vote prefs. over public goods

3. Sophisticated mechanisms to elicit true values
   → ~~E~~ e.g. Vickery - Groves - Clark (VGC) mechanism
   → way to elicit the Pareto efficient amount of public good, but doesn't ensure good provided in a Pareto efficient way

## More on voting

→ One problem: preferences of group as elicited as votes are not transitive

→ this leads to what are called Condorcet Cycles (or the Condorcet Paradox or the paradox of voting)

→ e.g. consider 3 voters, Alice, Becky, and Charlie

They have the following preference ordering for 3 Presidential candidates

| | Alice | Becky | Charlie |
|---|---|---|---|
| 1. | Trump | ~~Clinton~~ Cruz | ~~Trump~~ Clinton |
| 2. | Cruz | ~~Cruz~~ Clinton | ~~Clinton~~ Trump |
| 3. | Clinton | Trump | Cruz |

| Matchups | Winner |
|---|---|
| Trump v. Cruz | Trump (2-1) |
| Trump v. Clinton | Clinton (2-1) |
| ~~Trump~~ Cruz v. Clinton | Cruz (2-1) |

So if Do Trump v. Cruz, then winner against Clinton → get Clinton

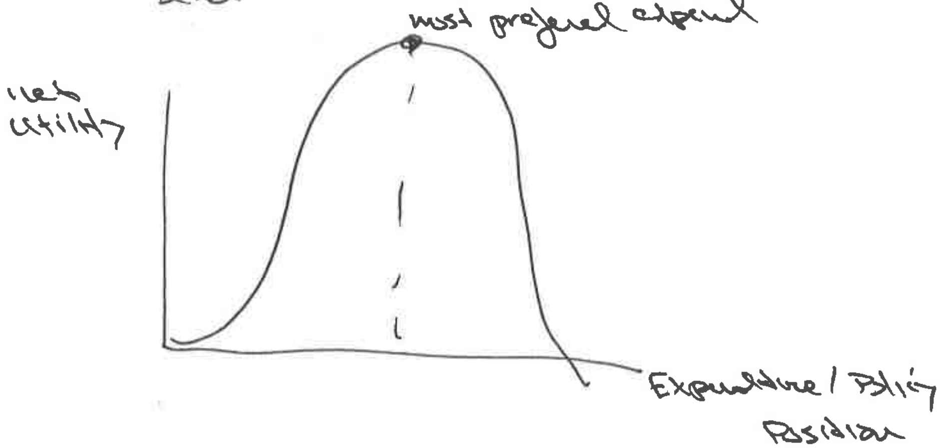
if Do Cruz v. Clinton, then winner against Trump → get Trump

if Do Trump v. Clinton, then winner against Cruz → get Cruz.

→ order of matchups determines outcome, not preferences of voters

→ something that would solve this intransitivity is if preferences were restricted in a certain way → in particular, if they were "single peaked"

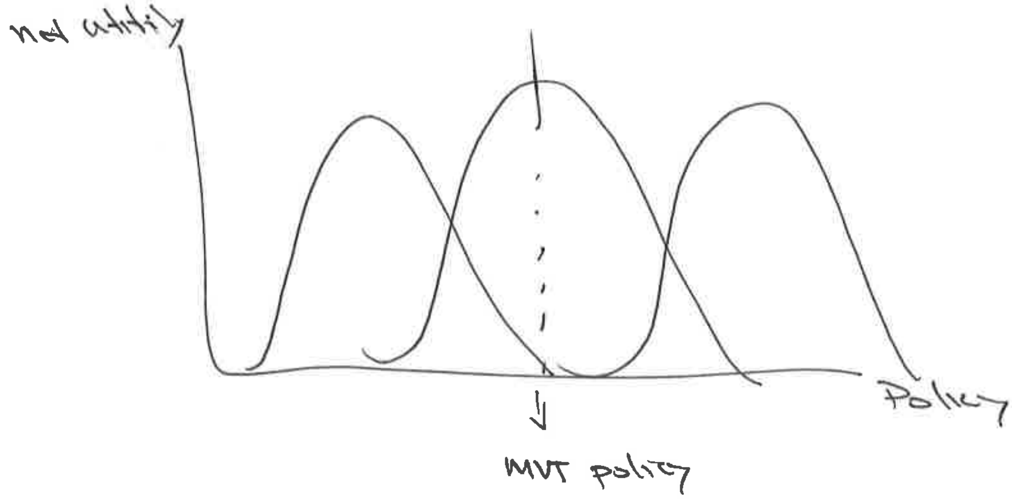
Single peaked preferences look like:




Not single-peaked


→ if preferences are single peaked, and the vote is over a single dimension of policy, then the most preferred policy of the voters will be that of the median voter

→ This is the result of the Median Voter Theorem (MVT)




→ eq'm policy is the most preferred point of the median voter
→ anything to the left loses to this
→ anything to the right loses to this

→ So the amount of public good provided?
It'll be the amount preferred by the median voter.
→ Half of the voters will want more, half less
→ Is this optimal? Generally, no.
→ why not?
→ votes can express more/less, but not how much more voters want the public good
→ b/c there is no measure of intensity, voting generally won't result in the Pareto efficient amount

Instrumental Voting

→ If people are voting to affect the outcome, we call this "instrumental voting"

→ But the value of instrumental voting depends on being the decisive (or pivotal) voter

→ if you don't cast the tie-breaking vote, your voter doesn't affect the outcome and so has no instrumental value

→ It's very unlikely you are the pivotal voter, so the instrumental value of voting is small

→ how small

→ consider an election where one candidate has a 51% change of ~~winn~~ winning and the other a 49% chance. There are 4 million voters

→ The probability of a tie is

$$\text{prob(tie)} = \frac{1}{\sqrt{\pi n}}(4p - 4p^2)^n$$

$$= \frac{1}{\sqrt{\pi\, 2{,}000{,}000}}(4(0.51) - 4(0.51^2))^{2{,}000{,}000}$$

$$= 0.0004\,\underbrace{(0.9996)^{2{,}000{,}000}}_{\sim 0}$$

= 1 in several billion

→ so little incentive to vote for what directly benefits you or be informed

→ But if no one informed, that may be a problem

→ Voting ~~in a pub~~lic turns every issue into a public good!

  → the private benefits to voting well are much less than the public benefits

But:

→ if everyone makes random mistakes, those should avg. out.

  → so we could be ok

→ But what if voters are biased w/o paying costs to becoming informed?

  → then we still have problems